Logic Machine

Introduction

In 1866, William Stanley Jevons, an English logician and economist constructed a machine known as the 'logical piano', which was capable of solving complicated problems with superhuman speed. This machine was inspired by George Boole's work on mathematical logic, known as 'Boolean Algebra'. Interestingly, a number of researchers in the 1930s noticed that the binary numbers (0 and 1), combined with the Boolean values of True and False, could be used to construct electrical switching circuits and thus used to design electronic computers. In this workshop we will introduce a variant of Jevons' logical piano, popularised by Martin Gardner in the mid-20th century, known as the 'the logical punch cards'. These punch cards combined logic with binary arithmetic and were widely used in the early computing era as they provided a convenient solution for storage, input and output for a computer.

Aim of Workshop

The aim of this workshop is to introduce students to the binary system and its connections to logic. The use of the logical punch cards will serve to demonstrate these principles in action and provide students with some historical context so that they can better appreciate the mathematics, ingenuity, and history behind a modern computer.

Learning Outcomes

By the end of this workshop, students will be able to:

- Construct a truth table to solve problems
- Explain, in their own words, how logical punch cards work
- Solve deductive reasoning problems using punch cards

Materials and Resources

A deck of 32 punch cards for each group of students (see appendix for template), activity sheets.

KEY WORDS

Binary
A system of counting which has '2' as its base rather than '10'. It is also commonly referred to as 'base 2'

Logic
The systematic study of the form of argument

Premise
A previous statement from which another follows as a conclusion

Logic Machine: Workshop Outline

Suggested Time (Total mins)	Activity	Description
10 mins (00:10)	Introduction to binary and logic	· Introduce the binary system and mention its importance in computer science (see **Appendix – Note 1)** · Draw connections between binary and Boolean algebra (see **Appendix – Note 2**) · Outline the historical background of Boolean algebra
15 mins (00:25)	Activity 1 Party invitations	· Introduce the concept of truth tables (see **Appendix – Note 3**) · **Activity Sheet 1:** Students complete Activity 1 using a truth table (see **Appendix – Note 4**)
10 mins (00:35)	Punch cards	· Demonstrate how to use punch cards to solve problems (see **Appendix – Note 5**)
15 mins (00:50)	Activity 2 Who is watching TV?	· **Activity Sheet 2:** In pairs or groups of three, students try to solve who is watching TV using the punch cards (see **Appendix – Note 6)**
10 mins (01:00)	Hamilton and Jevons	· Outline the work of Margaret Hamilton in the Apollo 11 mission (see **Appendix – Note 7**) · Mention William Stanley Jevons & his 'Logic piano' (see **Appendix – Note 8)**
10 mins (01:10)	Activity 3 Similar triangles (Optional)	· Explain the premises for the similar triangles puzzle (see **Appendix – Note 9**) · **Activity Sheet 3:** In pairs or groups of three, students attempt to solve the similar triangle puzzle using punch cards (see **Appendix – Note 10**)

Logic Machine: Workshop Appendix

Note 1: Introduction to Binary

Binary is a counting system based on successive powers of 2 and is made up of only two digits: 0 and 1. Gottfried Wilhelm Leibniz, a German mathematician, was the first person who studied the binary system in depth during the 17th century, solely for mathematical interest. Nowadays, however, binary is widely used in computer programming as it is the most efficient way to operate logic circuits. This is due to the fact that the digits 0 and 1 represent OFF and ON respectively and can, therefore, be used to control the state of electrical signals in a circuit.

Note: For more information on binary please refer to 'Base Systems', Maths Sparks Volume I.

Note 2: Introduction to Logic

Logic is an important concept that has applications in a wide variety of disciplines including computer science, linguistics, philosophy and mathematics. Whilst there are many different types of logic, this workshop will refer to Aristotelian logic, which concerns deductive reasoning as expressed in syllogisms e.g. All Greeks are men, all men are mortal, therefore all Greeks are mortal. However, we also draw on the work of George Boole, an English mathematician and first professor of mathematics at Queen's College, Cork (now UCC), who extended Aristotle's philosophical approach to logic and developed a mathematical system for interpreting logical statements. This later became known as 'Boolean algebra' and can be used to solve certain problems involving deductive reasoning.

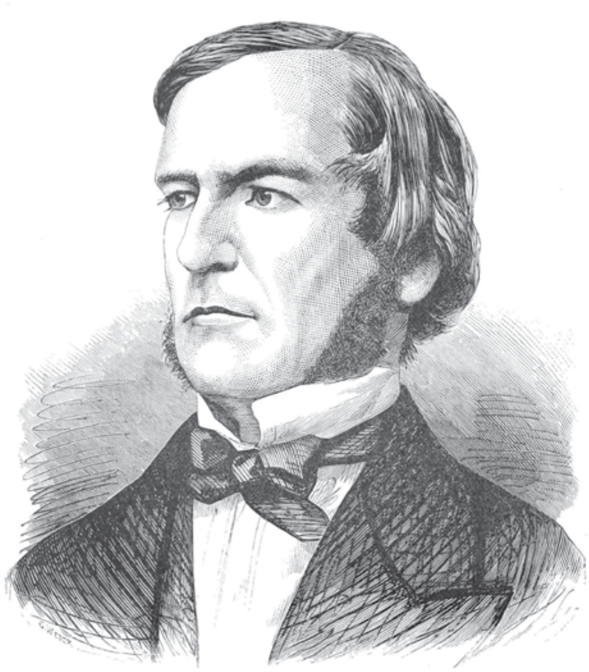


Figure 2: George Boole

Whilst the Greeks studied logic in the form of statements, Boolean algebra used the variables 'true' and 'false', which are commonly denoted by the binary digits '1' and '0' respectively. With this, Boole was able to condense a string of statements into a binary sequence. It is this connection between logic and binary that makes Boolean algebra so powerful in the digital world.

Note: For more information on logic please refer to 'Logic Gates', Maths Sparks Volume II.

Note 3: Truth Tables

A truth table is a convenient way of describing the outcomes of basic logic operations. They comprise a column for each variable, and rows that represent all possible situations (i.e. true and false combinations of our variables). We will use **Activity Sheet 1** to demonstrate how a truth table can be used to solve problems (see **Note 4**).

Note 4: Solutions for Activity 1

You have three friends: John, Paul, and Mary, whom you would like to invite to your party

1. If you invite Mary, you must invite John
2. If you don't invite Paul, you must invite both John and Mary
3. You must invite either John or Mary, but not both

Q1. Fill in the last two columns of the truth table below:

Row	John	Paul	Mary	Premise 1	Premise 2	Premise 3
i.	0	0	0	✔	X	X
ii.	0	0	1	X	X	✔
iii.	0	1	0	✔	✔	X
iv.	0	1	1	X	✔	✔
v.	1	0	0	✔	X	✔
vi.	1	0	1	✔	✔	X
vii.	1	1	0	✔	✔	✔
viii.	1	1	1	✔	✔	X

1 = invite, 0 = does not invite

Note: Each row in the table represents one scenario of who we invite to the party.

For example, row (i) represents all three people are not invited whereas row (iv) represents a situation where Paul and Mary are invited but John is not etc.

Since there are two choices for each person (invited or not invited), the total number of possible scenarios is $2 \times 2 \times 2 = 2^3 = 8$.

We can formulate the total number of combinations as follows:

Total number of combinations = 2^x, where x is the number of variables.

Since the truth table exhausts all possible scenarios, we can use it to eliminate combinations that do not satisfy our premises as illustrated above.

Q2. Based on the table above, who should you invite to the party?

You should invite John and Paul to the party but not Mary since this is the only combination that follows all three premises (see row (vii))

Note 5: Using Punch Cards

The method of using truth tables can become rather tedious as the number of variables increase, which, in turn, will also increase the number of possible combinations. This is where the punch card comes in (see Figure 3). The punch card works under the same principle as the truth table, with cards representing each of the rows of the truth table. The punch cards provide a better method of filtering and removing contradictory combinations and save time when we have a large number of variables. For **Activity Sheet 1,** we would use 8 individual punch cards with the last three slots on each card denoting the three variables (i.e. John, Paul and Mary).

In order to demonstrate how punch cards work, we will use the same example as outlined in **Activity Sheet 1**. However, we will use the following abbreviations to simplify our explanation:

$\text{J} = \text{John is invited}$

$\bar{\text{J}} = \text{John is not invited (read as "J bar")}$

$\text{P} = \text{Paul is invited}$

$\bar{\text{P}} = \text{Paul is not invited (read as "P bar")}$

$\text{M} = \text{Mary is invited}$

$\bar{\text{M}} = \text{Mary is not invited (read as "M bar")}$

Figure 3: Example of a punch card

Premise 1: If you invite Mary, you must invite John

Based on the first premise, we are looking to eliminate combinations where Mary is invited but John is not invited, i.e. eliminate combinations with $\overline{\text{J}}\,\text{M}$. As shown in the truth table, we can eliminate row (ii) and row (iv) which do not satisfy the first premise. The converse, however, where John is invited but Mary is not invited might or might not be true, so we can keep those combinations.

	John is invited	Paul is invited	Mary is invited	Keep/discard
i.	0	0	0	Keep
ii.	0	0	1	Discard
iii.	0	1	0	Keep
iv.	0	1	1	Discard
v.	1	0	0	Keep
vi.	1	0	1	Keep
vii.	1	1	0	Keep
viii.	1	1	1	Keep

To eliminate the contradictory combinations (row (ii) and (iv)) using the punch cards, we first insert a pen (or skewer) into the slot labelled 'Mary' and lift the cards. As the cards that correspond to M (Mary is invited) have an open slit, they will not be lifted. Whereas cards that correspond to $\bar{\mathrm{M}}$ (Mary is not invited) will not have an open slit, hence they will be lifted and stay on the pen (see Figure 4). The deck is now sorted into 2 piles of cards:

Pile $\bar{\mathrm{M}}$	Mary is not invited
Pile M	Mary is invited

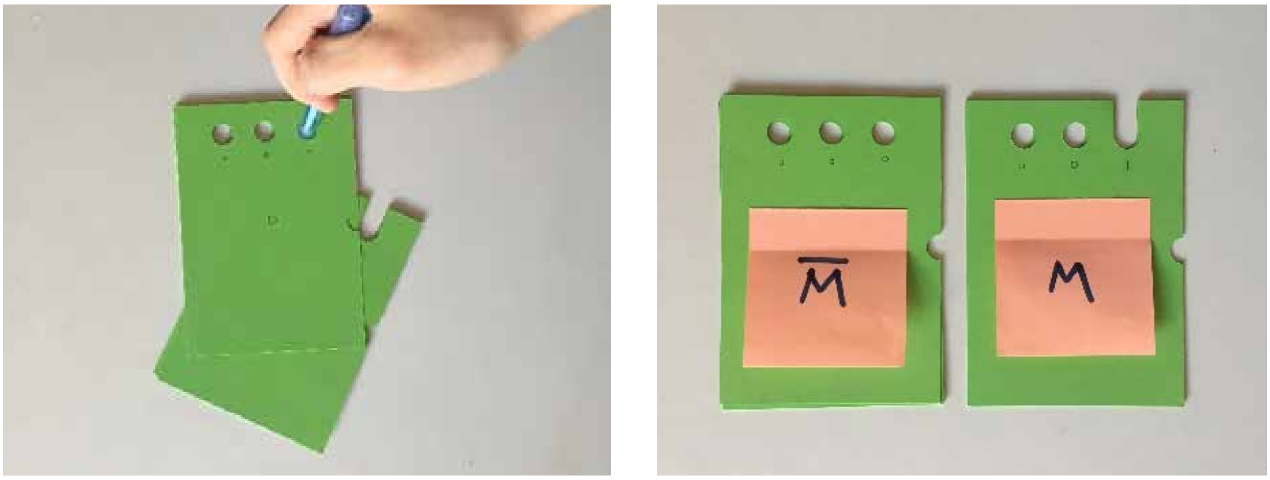


Figure 4: Filtering combinations where Mary is invited

From Pile M, we need to further separate combinations into piles where John is not invited, and John is invited. We achieve this by inserting a pen into the slot labelled John and lifting the cards. As before, the cards without a slit will be lifted and the rest will be left behind. This will result in two more piles:

Pile $\bar{\mathrm{M}}$	Mary is not invited
Pile $\bar{\mathrm{J}}\mathrm{M}$	John is not invited but Mary is invited
Pile JM	John and Mary both not invited

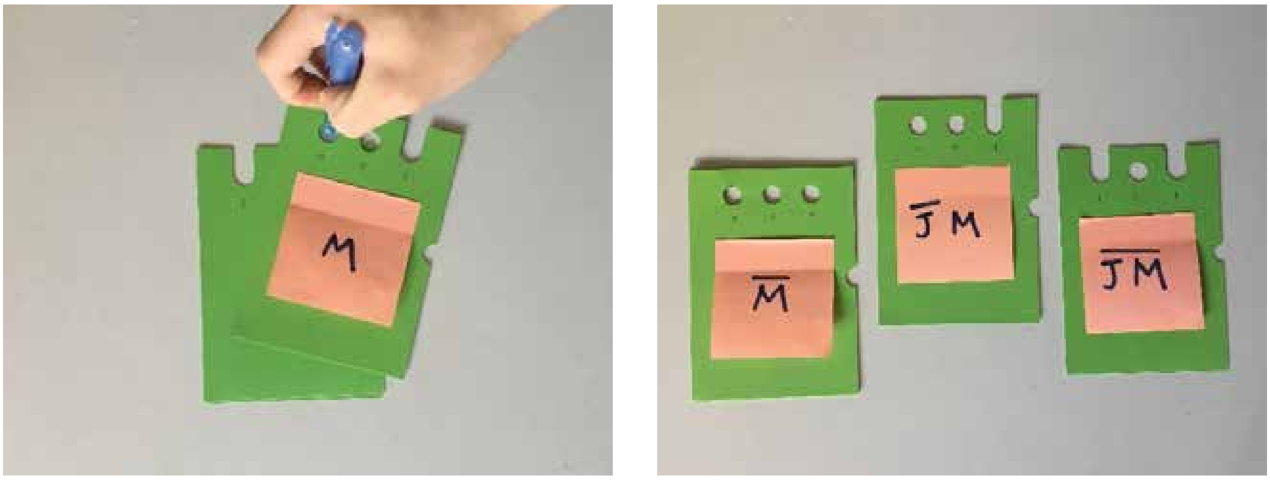


Figure 5: Filtering combinations where Mary and John are invited

We discard Pile $\bar{J}M$ because this pile does not satisfy premise 1. We can now combine Pile $\bar{M}$ with Pile JM and proceed to the next premise (Note: the discarded cards are no longer needed).

Premise 2: If you don't invite Paul, you must invite both John and Mary.

As the premise suggests, we only keep the combination where Paul is not invited but both John and Mary are invited (row vi). Since this premise does not concern what happens when Paul is invited, we can keep the rows with a 1 in the second column since they are still valid (i.e. row (iii), (iv), (vii), and (viii)). Note: the red or struck through lines signify the piles we discarded based on the first premise and are therefore no longer valid.

	John is invited	Paul is invited	Mary is invited	Keep/discard
i.	0	0	0	Keep
~~ii.~~	~~0~~	~~0~~	~~1~~	~~Discard~~
iii.	0	1	0	Keep
~~iv.~~	~~0~~	~~1~~	~~1~~	~~Discard~~
v.	1	0	0	Keep
vi.	1	0	1	Keep
vii.	1	1	0	Keep
viii.	1	1	1	Keep

Using the punch cards, we separate $\bar{P}$ with P using a pen and the slot labelled P. We do not discard pile P since the premise says nothing about what happens if Paul is invited.
(Note: Since pile $\bar{P}$ only contains three cards ($\overline{JPM}$, $J\overline{PM}$ and $J\bar{P}M$), it is possible to go through all three cards and select the one that does not contradict the premise ($J\bar{P}M$). However, we will outline the formal approach below since this method would be tedious for a larger sample of punch cards).

Pile $\bar{P}$	Paul is not invited
Pile P	Paul is invited

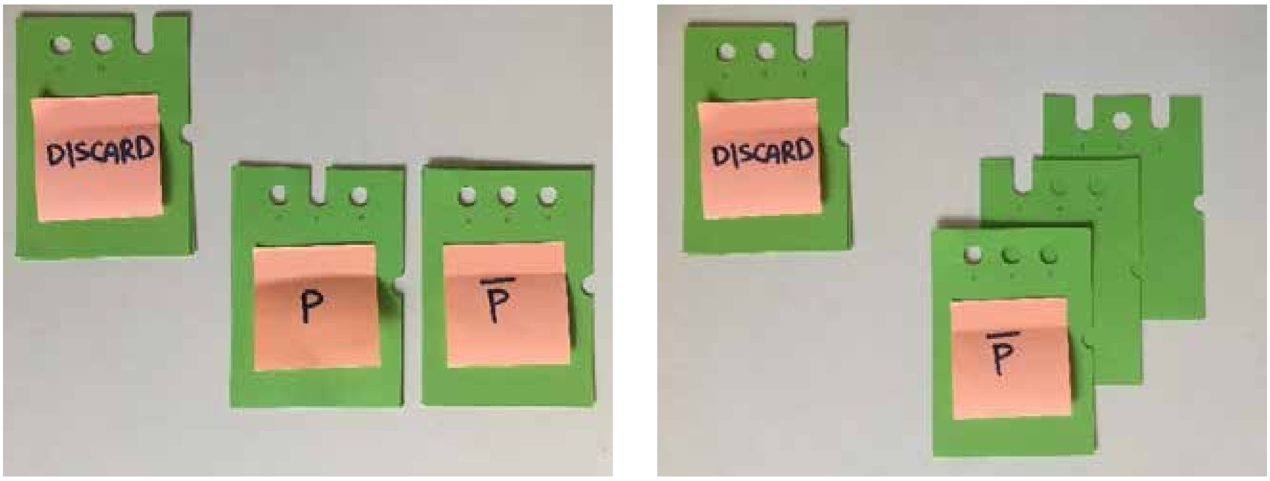


Figure 6: Filtering combinations where Paul is not invited

Once we have identified the pile $\bar{P}$ (Paul is not invited), we further divide pile $\bar{P}$ into two piles:

Pile $J\bar{P}$	John is invited when Paul is not invited
Pile $\overline{JP}$	John is not invited when Paul is not invited

Figure 7: Filtering combinations where John is invited when Paul is not invited

Based on the second premise, we must invite John, so we can discard pile $\overline{J}\overline{P}$. We then separate pile $J\overline{P}$ into two more piles:

Pile $J\overline{PM}$	John is invited but Mary is not invited when Paul is not invited
Pile $J\overline{P}M$	John and Mary are invited when Paul is not invited

We choose to keep $J\overline{P}M$ and discard $J\overline{PM}$ We can now recombine this card with the pile P as this premise does not say anything about situations where P is invited so this pile is still valid.

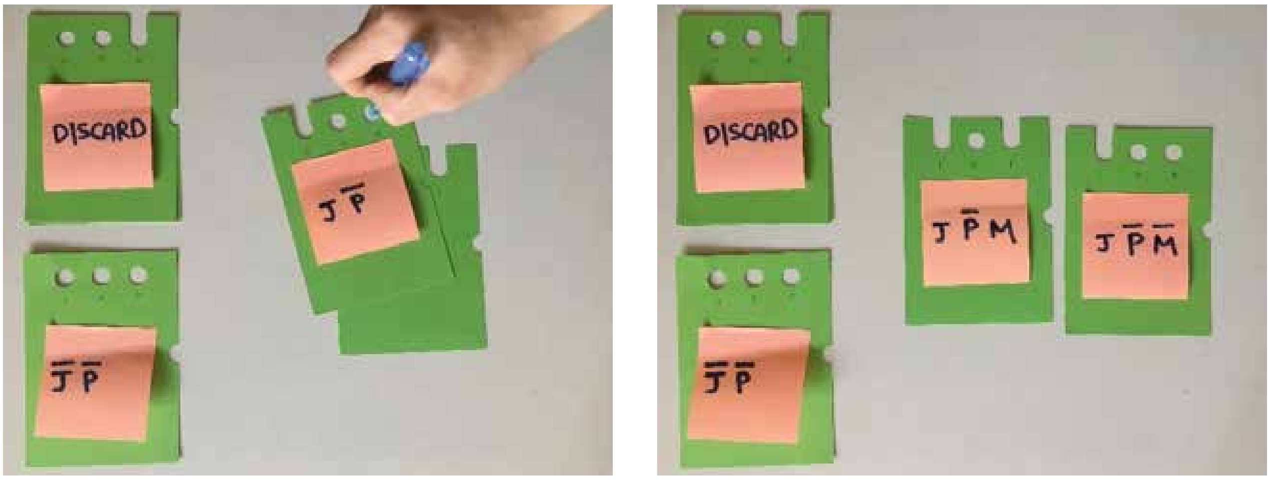


Figure 8: Filtering combinations where John and Mary are invited when Paul is not invited

Premise 3: You must invite either John or Mary, but not both.

Based on this final premise we want to eliminate combinations where John and Mary are both invited, i.e. eliminate JM. So, we discard row (vii) and (viii). Since, the term 'either' was used, it means that we should at least invite one of John and Mary. Hence, we can eliminate the combination where John and Mary are both uninvited, i.e. eliminate $\overline{JM}$. This leaves us with one final combination which brings us the answer that we should invite John and Paul. Note: the red or struck through rows in the table signify the piles we already discarded based on both the first and second premise and are therefore no longer in the pile.

	John is invited	Paul is invited	Mary is invited	Keep/discard
~~i.~~	~~0~~	~~0~~	~~0~~	~~Discard~~
ii.	~~0~~	~~0~~	~~1~~	~~Discard~~
iii.	0	1	0	Discard
~~iv.~~	~~0~~	~~1~~	~~1~~	~~Discard~~
~~v.~~	~~1~~	~~0~~	~~0~~	~~Discard~~
vi.	1	0	1	Discard
vii.	1	1	0	Keep
viii.	1	1	1	Discard

With the punch cards, we do this by first diving the deck into two piles using the first slot to obtain:

Pile $\bar{J}$	John is not invited
Pile J	John is invited

Then, from each of the piles, we separate M with $\bar{M}$ using the third slot to obtain four piles:

Pile $\bar{J}M$	John is not invited but Mary is invited
Pile $\overline{JM}$	John and Mary both not invited
Pile $J\bar{M}$	John is invited but Mary is not
Pile JM	John and Mary are both invited

Finally, we discard pile $\bar{J}\bar{M}$ and JM. At this point, there should be only one card left: $J\bar{P}\bar{M}$, which is the combination that does not contradict any of the three premises above.

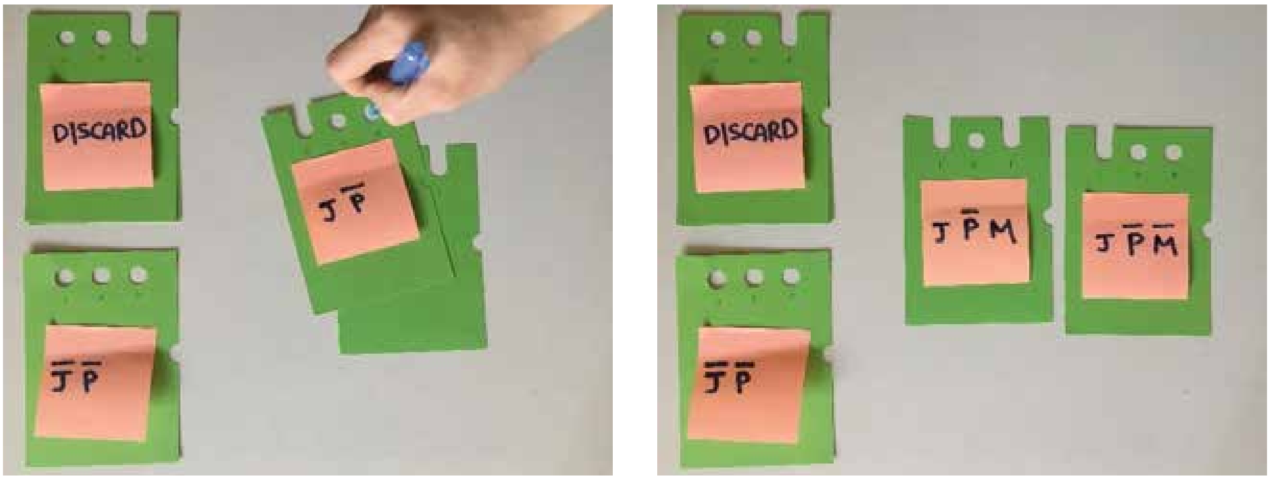


Figure 9: Filtering combinations where either John or Mary are invited but not both

Note: For a deck of 32 punch cards with 5 slots (instead of 3), you could divide the deck into 4 smaller decks: 0-7, 8-15, 16-23, and 24-31, based on the numbering on the cards and ignore the first two slots.

Note 6: Solutions for Activity 2

Who is watching television?

Albert and his wife Breda have three children: Ciara, David and Ellie.

1. If Albert is watching television, so is his wife.
2. Either David or Ellie, or both of them, are watching television.
3. Either Breda or Ciara, but not both, is watching television.
4. David and Ciara are either both watching television or both not watching television.
5. If Ellie is watching television, then Albert and David are also watching

Note: Each group of students will need the full set of 32 punch cards for this activity.

Q1. How many possible scenarios (combination of who is watching and who is not watching television) could we have for this TV watching session?

Since there are 5 people, each person could either be watching TV or not watching TV.

Thus, the number of possible scenarios is $2^5 = 32$

Q2. From the first premise, which combination should we eliminate?

AB – Albert and Breda both watching TV.

$\mathrm{A\bar{B}}$ – Albert is watching TV and Breda is not watching TV.

$\mathrm{\bar{A}B}$ – Breda is watching TV and Albert is not watching TV.

$\overline{\mathrm{AB}}$ – Albert and Breda both not watching TV.

The correct answer is $\mathrm{A\bar{B}}$. Students might also have $\mathrm{\bar{A}B}$ as an answer but the statement makes no assumption about what Albert will do if Breda watches television.

Q3. How could you remove the contradictory combination from Q2 from the deck? Describe Briefly.

- Using a set of 32 cards, label the five slots A, B, C, D and E respectively.
- To filter $\mathrm{A\bar{B}}$, we first separate cards of A with $\mathrm{\bar{A}}$ by inserting a pencil into the first slot and lifting the cards up. The cards that were left behind are the A pile (Albert is watching TV) while the cards that were lifted are the $\mathrm{\bar{A}}$ (Albert is not watching TV).
- From the A pile, we insert a pencil into the second slot and further separate the A pile into $\mathrm{A\bar{B}}$ pile (hang on the pencil) and AB pile (left behind).
- Discard the $\mathrm{A\bar{B}}$ pile and recombine the $\mathrm{\bar{A}}$ pile and AB pile. You should be left with 24 cards.

Q4. From the second premise, what combinations should we eliminate?

David is watching TV	Ellie is watching TV	Keep/discard
0	0	Discard
0	1	Keep
1	0	Keep
1	1	Keep

Based on the table above, we should eliminate the combination $\bar{D}\bar{E}$ (David and Ellie both are not watching TV). The combination $\bar{D}\bar{E}$ contradicts the premise because the premise uses the term 'either', therefore we need to have at least one of them (David or Ellie) watching TV.

Q5. How could you remove the contradictory combination from Q4 from the deck?

- To filter out $\bar{D}\bar{E}$, we first separate D from $\bar{D}$ using the fourth slot
- Then further divide the $\bar{D}$ pile into $\bar{D}E$ and $\bar{D}\bar{E}$ piles using the fifth slot, discard the $\bar{D}\bar{E}$ pile and recombine the rest. You should be left with 18 cards at this point

Q6. Based on the third premise, what combinations should we eliminate?

Breda is watching TV	Ciara is watching TV	Keep/discard
0	0	Discard
0	1	Keep
1	0	Keep
1	1	Discard

Based on the table above, there are two combinations that should be eliminated, $\bar{B}\bar{C}$ (Breda and Ciara both are not watching TV) and BC (Breda and Ciara both are watching TV).

Q7. How could you remove the contradictory combinations from Q6 from the deck?

Note: This premise involves removing two combinations, $\bar{B}\bar{C}$ and BC.

- We first separate B with $\bar{B}$ using the second slot
- Further divide both piles separately using the third slot to obtain four piles: BC, $B\bar{C}$, $\bar{B}C$, and $\bar{B}\bar{C}$. Discard the $\bar{B}\bar{C}$ and BC piles. Recombine the rest. You should be left with 9 cards

Q8. Based on the fourth premise, what combinations should we eliminate?

Ciara is watching TV	David is watching TV	Keep/discard
0	0	Keep
0	1	Discard
1	0	Discard
1	1	Keep

Based on the table above, the two combinations that should be eliminated are $C\bar{D}$ (Ciara is watching TV and David is not watching TV) and $\bar{C}D$ (David is watching TV and Ciara is not watching TV).

Q9. How could you remove the contradictory combinations from Q8 from the deck?

To remove the combination $C\bar{D}$ and $\bar{C}D$, we repeat a similar process as Q7, by first dividing the deck into C and $\bar{C}$ piles (using the third slot), then further divide both piles separately into CD, $\bar{C}D$, $C\bar{D}$ and $\bar{C}\bar{D}$ piles (using the fourth slot). Discard $C\bar{D}$ and $\bar{C}D$ piles and recombine the rest. You should be left with 4 cards at this point.

Q10. Based on the final premise, what combinations should we eliminate?

The premise only concerns cases where Ellie is watching TV, therefore we keep all the combinations with $\bar{E}$ (Ellie is not watching TV) and only focus on cases with E (Ellie is watching TV).

Ellie is watching TV	Albert is watching TV	David is watching TV	Keep/discard
1	0	0	Discard
1	0	1	Discard
1	1	0	Discard
1	1	1	Keep

Based on the table above, we need to discard three combinations: $E\bar{A}\bar{D}$, $EA\bar{D}$, and $E\bar{A}D$.

Q11. How could you remove the contradictory combinations from Q10 from the deck?

As there are only 4 cards left, it is possible to discard cards with the contradictory combinations by going through each of them. This will eliminate every card with the binary representation $0xx01$, $0xx11$, and $1xx01$.

Another method is to separate the deck into E and $\bar{E}$ pile (using the fifth slot). Since we are eliminating three combinations and only keeping one, it is faster to extract the combination we would like to keep: EAD.

You may notice that you cannot extract cards with combination EAD since these cards were removed by previous premises. Meaning that there are no cards in the $\bar{E}$ pile that satisfy the premise. You can then discard the $\bar{E}$ pile and you will be left with one card: $\bar{A}\bar{B}CD\bar{E}$.

Q12. Can you now determine who is watching TV?

Since the final punch card is $\bar{A}\bar{B}CD\bar{E}$, we now know that only Ciara and David are watching TV. This is the only combination/scenario that satisfies all five premises.

Note: It is possible to go through the premises in any order and with the cards shuffled; the result should be the same. Below is a table of cards that should be eliminated during each of the premises.

A	B	C	D	E	Premise 1	Premise 2	Premise 3	Premise 4	Premise 5
0	0	0	0	0		x	x		
0	0	0	0	1			x		x
0	0	0	1	0			x	x	
0	0	0	1	1			x	x	x
0	0	1	0	0		x		x	
0	0	1	0	1				x	x
0	0	1	1	0					
0	0	1	1	1					x
0	1	0	0	0		x			
0	1	0	0	1					x
0	1	0	1	0				x	
0	1	0	1	1				x	x
0	1	1	0	0		x	x	x	
0	1	1	0	1			x	x	x
0	1	1	1	0			x		
0	1	1	1	1			x		x
1	0	0	0	0	x	x	x		
1	0	0	0	1	x		x		x
1	0	0	1	0	x		x	x	
1	0	0	1	1	x		x	x	
1	0	1	0	0	x	x		x	
1	0	1	0	1	x			x	x
1	0	1	1	0	x				
1	0	1	1	1	x				
1	1	0	0	0		x			
1	1	0	0	1					x
1	1	0	1	0				x	
1	1	0	1	1				x	
1	1	1	0	0		x	x	x	
1	1	1	0	1			x	x	x
1	1	1	1	0			x		
1	1	1	1	1			x		

Note 7: Margaret Hamilton

Margaret Hamilton is an American computer scientist and systems engineer who led the MIT Software Engineering Division, which was responsible for developing the computer systems on board the Apollo 11 spacecraft. Hamilton and her team used a more robust version of punch cards known as core rope memory to programme these computers, which like all modern computers, stored information in binary arithmetic.

Note 8: William Stanley Jevons and the Logic Piano

William Stanley Jevons was an English economist and logician who is credited with the invention of the 'logic piano'; the first logic machine to solve complicated problems at an exceptional speed. Much like a musical piano, this device consisted of a series of black-and-white keys, which were used for entering premises. A face plate was placed above the keyboard and displayed the truth table. As the keys were struck, rods would mechanically remove the truth table entries inconsistent with the premises entered on the keys. The logical punch cards are a variant of the logical piano and were popularised by Martin Gardner in the mid-20th century.

Note 9: Premises for Activity 3

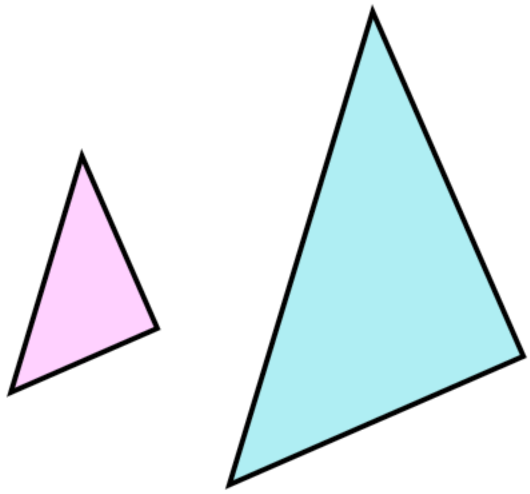
The problem below was obtained from George Boole's first publication on the theory of Boolean Algebra in 1854, which was entitled 'An Investigation of the Law of Thought'. In the book, Boole solves this problem using pen-and-paper (Boolean) algebra. However, the punch cards and truth tables use a similar principle.

Premise 1: Triangles whose corresponding angles are equal have their corresponding sides proportional, and vice versa.

The first premise states a property of triangles:

- Two triangles have the same angle at each of their corresponding vertices, *if and only if* (⟺) the corresponding sides of the two triangles are proportional.

Note: This statement does not mention anything about similarity.

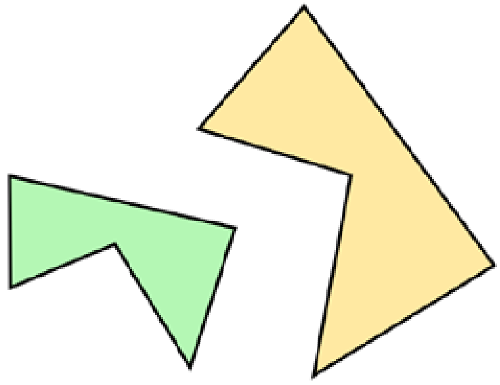


Premise 2: Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.

The second premise is a definition of what it means for two figures (polygons) to be similar:

- All the corresponding vertices have the same angle and
- All the corresponding sides are proportional.

Note: This statement does not mention anything about triangles.



You may wish to ask the students if the first statement still holds for shapes other than triangles (One counterexample would be a square and a rectangle, they have equal angles at every vertex (90°), but the sides are not proportional).

Using the following premises:

1. Triangles whose corresponding angles are equal have their corresponding sides proportional, and vice versa.
2. Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.

Which statement is true?

A. Triangles, for which their corresponding sides are proportional or corresponding angles are equal (but not both) are similar triangles.

B. Dissimilar triangles, for which their corresponding angles are equal, have corresponding sides that are disproportional.

C. Figures, for which their corresponding sides are proportional, and their corresponding angles are equal (but are not triangles) are similar.

D. Non-triangular, dissimilar figures could have corresponding sides proportional, and corresponding angles equal.

Note 10: Solutions for Activity 3

Note: This problem only involves four terms (S, T, Q and R), which gives a total of 16 possible combinations. You could, therefore, split a full deck of 32 punch cards into two smaller decks (one deck with 0 - 15 and another with 16 - 31) as this would allow for twice the number of student groups.

To represent these premises, we can let:

S = similar

T = triangles

Q = having corresponding angles equal

R = having corresponding sides proportional

Q1. From the second premise, which combinations should we eliminate?

Since the premise only makes a statement about triangles, we focus on cases with $\overline{T}$.

T	Q	R	Keep/discard
1	0	0	Keep
1	0	1	Discard
1	1	0	Discard
1	1	1	Keep

Any triangle that satisfies Q must also satisfy R and vice versa. Thus, we eliminate $T\bar{Q}R$ and $TQ\bar{R}$.

Q2. How could you remove the contradictory combinations from Q1 from the deck?

Labelling four slots with S, T, Q, and R. We first separate T with $\bar{T}$ (using the T slot). From the T pile, separate Q with $\bar{Q}$ (using the slot Q) to get two piles: TQ and $T\bar{Q}$. Further divide TQ and $T\bar{Q}$ (using the slot R) into four piles: TQR, $TQ\bar{R}$, $T\bar{Q}R$, and $T\bar{Q}\bar{R}$. Discard $TQ\bar{R}$ and $T\bar{Q}R$ then recombine the rest. You should be left with 12 cards.

Q3. From the second premise, which combinations should we eliminate?

S	Q	R	Keep/discard
0	0	0	Keep
0	0	1	Keep
0	1	0	Keep
0	1	1	Discard
1	0	0	Discard
1	0	1	Discard
1	1	0	Discard
1	1	1	Keep

We eliminate $S\bar{Q}\bar{R}$, $S\bar{Q}R$, and $SQ\bar{R}$ because both conditions (Q and R) need to be satisfied for two figures to be similar. We also eliminate $\bar{S}QR$ because if both conditions (Q and R) are satisfied, the two figures are similar by definition.

Q4. How could you remove the contradictory combinations from Q3 from the deck?

First, we separate S from $\bar{S}$ (using the S slot). From the S pile, we would like to keep SQR. We first divide S into SQ and $S\bar{Q}$ (using Q slot), then further divide SQ into SQR and $SQ\bar{R}$ (using R slot). Keep the SQR pile and discard the rest.

Meanwhile, from the $\bar{S}$ pile, we would like to eliminate $\bar{S}QR$. We achieve this by separating the $\bar{S}$ pile into $\bar{S}Q$ and $\bar{S}\bar{Q}$ (using slot Q), then further divide $\bar{S}Q$ into $\bar{S}QR$ and $\bar{S}Q\bar{R}$ (using slot R). Discard $\bar{S}QR$ and recombine $\bar{S}\bar{Q}$, $\bar{S}Q\bar{R}$ and SQR from above.

You should be left with 6 cards at this point. These are $\bar{S}\bar{T}\bar{Q}\bar{R}$, $\bar{S}\bar{T}\bar{Q}R$, $\bar{S}\bar{T}Q\bar{R}$, $\bar{S}T\bar{Q}\bar{R}$, $S\bar{T}QR$, and $STQR$ respectively. These are all the combinations that do not contradict the two premises we have.

Q5. Express each of the options (A, B, C, and D) of the question in terms of $S, \bar{S}, T, \bar{T}, Q, \bar{Q}, R, \bar{R}$. Which statement is valid?

Each of the statements could be expressed as follows:

A. $T\bar{Q}R$ and $TQ\bar{R}$

B. $\bar{S}TQ\bar{R}$

C. $S\bar{T}QR$

D. $\bar{S}\bar{T}QR$

The only valid statement is C ($S\bar{T}QR$) as it is one of the six cards that does not contradict the two premises.

A and B are invalid because any triangle that satisfies Q must also satisfy R and vice versa.

D is invalid because figures that satisfy Q and R, by definition, are similar.

Remark: Below is a table of cards that should be eliminated during each of the premises

S	T	Q	R	Premise 1	Premise 2
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		X
0	1	0	0		
0	1	0	1	X	
0	1	1	0	X	
0	1	1	1		X
1	0	0	0		X
1	0	0	1		X
1	0	1	0		X
1	0	1	1		
1	1	0	0		X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1		

X represents cards that contradict the premise.

Sources and Additional Resources

Boole, G (1854). *An Investigation of the Law of Thought. Open Court.*

Gardner, M. (1969). Martin Gardner's new mathematical diversions from 'Scientific American'. London: Allen & Unwin.

http://www.upriss.org.uk/db/slides/logicpuzzle.pdf (Party invitation problem)

http://assets.cambridge.org/97805217/56075/excerpt/9780521756075_excerpt.pdf (Who is watching TV?)

http://history-computer.com/ModernComputer/thinkers/Jevons.html (Logic piano)

https://airandspace.si.edu/stories/editorial/rope-mother-margaret-hamilton (Margaret Hamilton)

Logic Machine: Activity 1

You have three friends John, Paul, and Mary, whom you would like to invite to your party.

However, there are a few premises to consider:

1. If you invite Mary, you must invite John.
2. If you don't invite Paul, you must invite both John and Mary.
3. You must invite either John or Mary, but not both.

The Party Invitation

Q1. Fill in the last two columns of the truth table below by writing ' ✕ ' to indicate combination that contradicts the premise and ' ✓ ' to indicate combination that does not contradict the premise in the respective column.

John	Paul	Mary	Premise 1	Premise 2	Premise 3
0	0	0	✓		
0	0	1	✕		
0	1	0	✓		
0	1	1	✕		
1	0	0	✓		
1	0	1	✓		
1	1	0	✓		
1	1	1	✓		

1 = invite, 0 = does not invite

Q2. Based on the table above, who should you invite to the party?

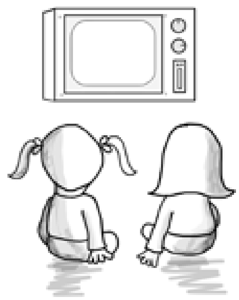
Logic Machine: Activity 2

Albert and his wife Breda have three children: Ciara, David and Ellie.

1. If Albert is watching television, so is his wife.
2. Either David or Ellie, or both of them, are watching television.
3. Either Breda or Ciara, but not both, is watching television.
4. David and Ciara are either both watching television or both not watching television.
5. If Ellie is watching television, then Albert and David are also watching

Who is watching TV?

Q1. How many possible scenarios (combinations of who is watching and who is not watching television) could we have for this TV watching session? How many cards do you need?



Q2. From the first premise, which combination should we eliminate?
(Hint: does the statement say anything about Albert if Breda is watching television?)

AB – Albert and Breda both watching TV.

$\mathrm{A\bar{B}}$ – Albert is watching TV and Breda is not watching TV.

$\mathrm{\bar{A}B}$ – Breda is watching TV and Albert is not watching TV.

$\mathrm{\bar{A}\bar{B}}$ – Albert and Breda both not watching TV.

Q3. How could you remove the contradictory combination from Q2 from the deck?
You may wish to discuss this with your group.

Q4. From the second premise, what combinations should we eliminate? (Write ' ✕ ' or ' ✓ ' in the last column to indicate discard or keep respectively).

David is watching TV	Ellie is watching TV	Keep/discard
0	0	
0	1	
1	0	
1	1	

Q5. How could you remove the contradictory combination from Q4 from the deck? You may wish to discuss this with your group.

Q6. Based on the third premise, what combinations should we eliminate? (Hint: There are multiple combinations that should be eliminated).

Breda is watching TV	Ciara is watching TV	Keep/discard
0	0	
0	1	
1	0	
1	1	

Q7. How could you remove the contradictory combinations from Q6 from the deck? You may wish to discuss this with your group.

Q8. Based on the fourth premise, what combinations should we eliminate?

Ciara is watching TV	David is watching TV	Keep/discard
0	0	
0	1	
1	0	
1	1	

Q9. How could you remove the contradictory combinations from Q8 from the deck? You may wish to discuss this with your group.

Q10. Based on the final premise, what combinations should we eliminate? (Beware that this premise involves 3 people).

Ellie is watching TV	Albert is watching TV	David is watching TV	Keep/discard
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Q11. How could you remove the contradictory combinations from Q10 from the deck? You may wish to discuss this with your group.

Q12. Can you now determine who is watching tv?

Logic Machine: Activity 3

Using the following premises:

1. Triangles whose corresponding angles are equal have their corresponding sides proportional, and vice versa.
2. Similar figures consist of all whose corresponding angles are equal, and whose corresponding sides are proportional.

Which statement is true?

A. Triangles, for which their corresponding sides are proportional or corresponding angles are equal (but not both) are similar triangles.

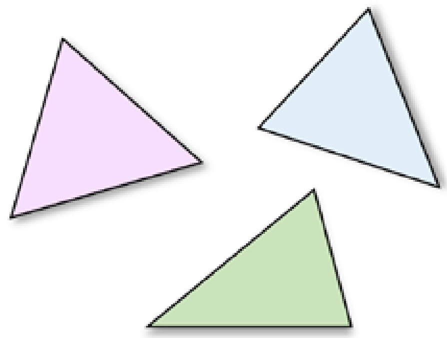
B. Dissimilar triangles, for which their corresponding angles are equal, have corresponding sides that are disproportional.

C. Figures, for which their corresponding sides are proportional, and their corresponding angles are equal (but are not triangles) are similar.

D. Non-triangular, dissimilar figures could have corresponding sides proportional, and corresponding angles equal.

To represent these premises, we can let

S	=	similar
T	=	triangles
Q	=	having corresponding angles equal
R	=	having corresponding sides proportional



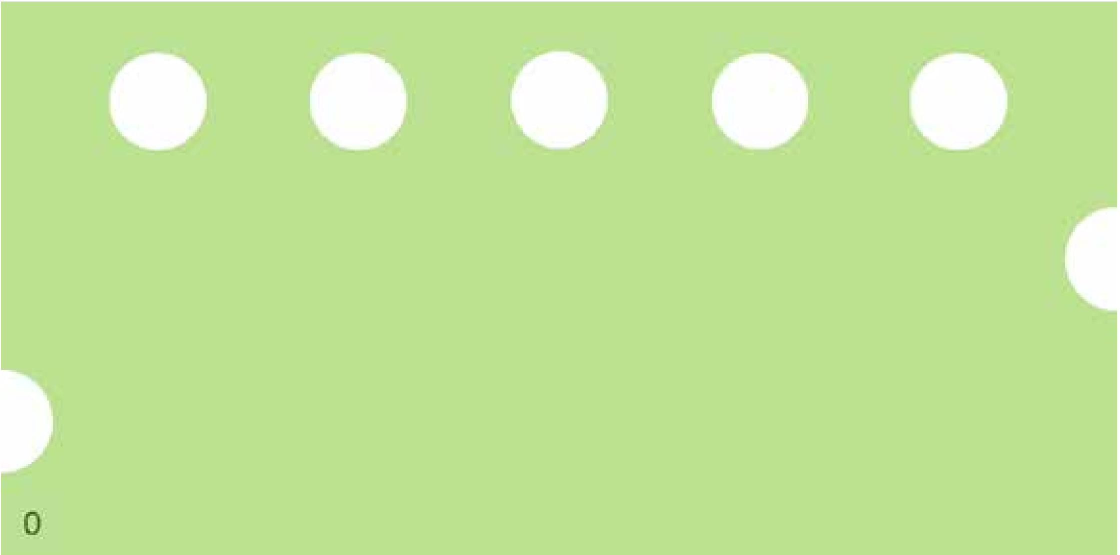
Q1. From the second premise, which combinations should we eliminate? (there is more than one combination to be eliminated)

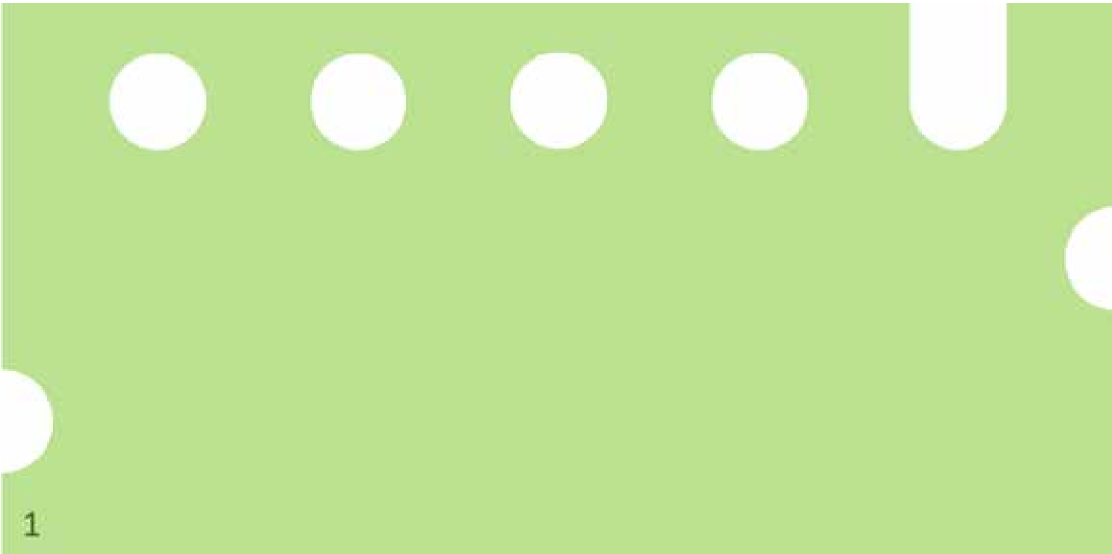
Q2. How could you remove the contradictory combinations from Q1 from the deck?

Q3. From the second premise, which combinations should we eliminate? (there is more than one combination to be eliminated).

Q4. How could you remove the contradictory combinations from Q3 from the deck?

Q5. Express each of the options (A, B, C, and D) of the question in terms of $S, \bar{S}, T, \bar{T}, Q, \bar{Q}, R, \bar{R}$. Which statement is valid?


0


1


2


3


4


5


6


7


8


9


10

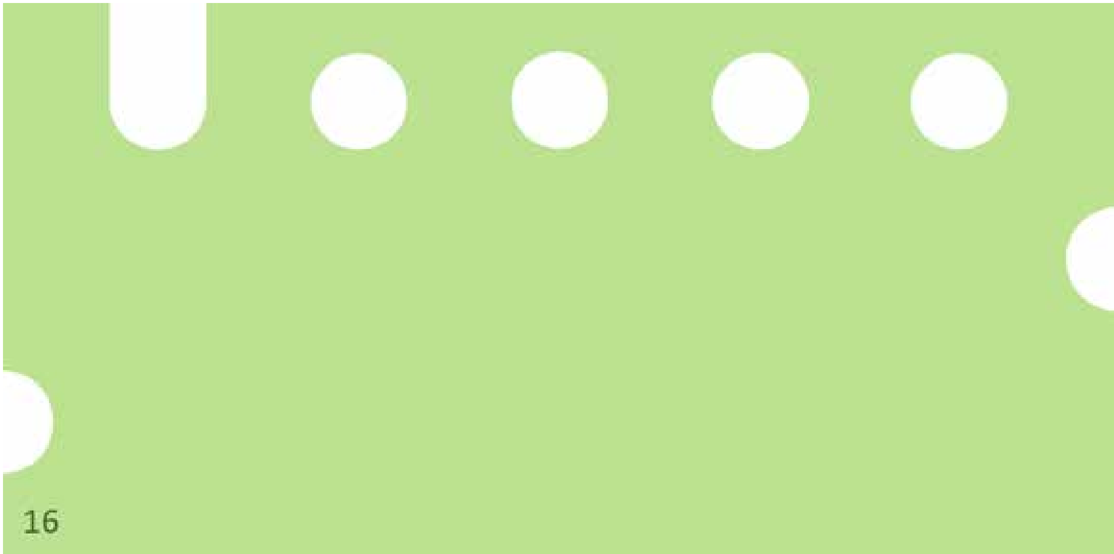

11

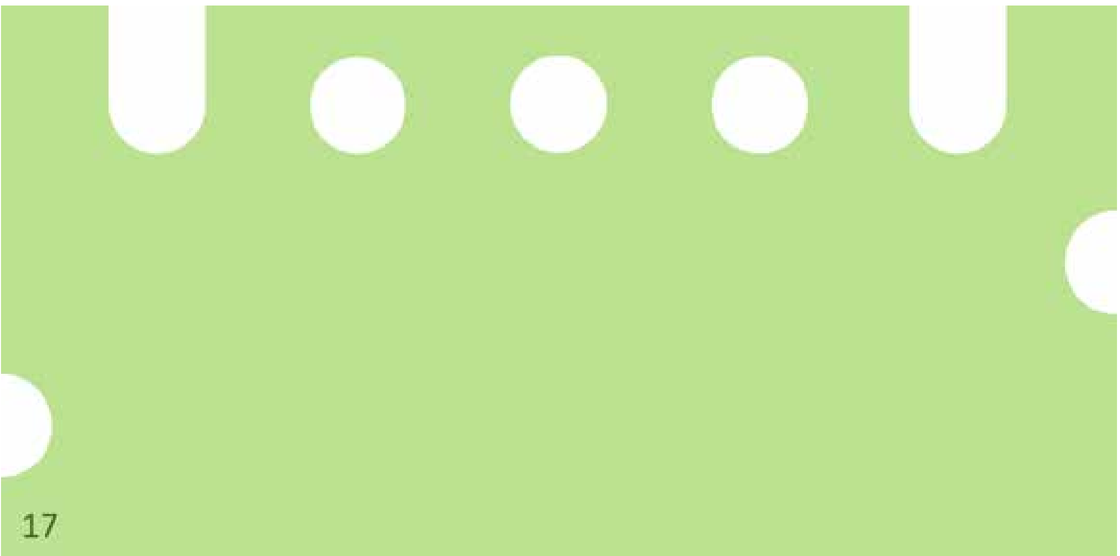

12


13

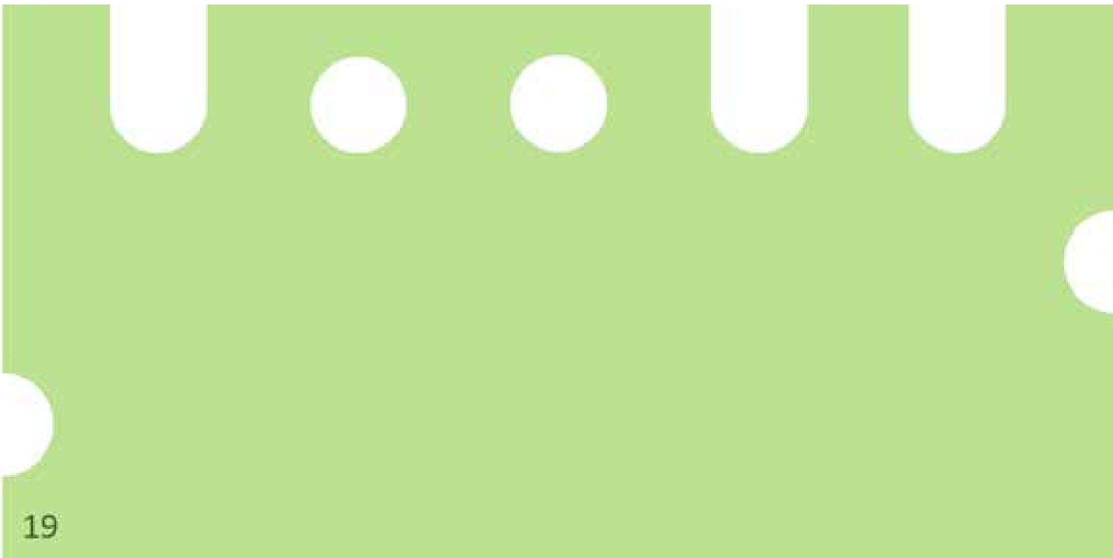

14


15


16


17


18


19


20


21


22


23


24


25


26


27


28


29


30


31